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A revised paper:

LOUDSPEAKER MINIMUM PHASE ESTIMATION

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ABSTRACT - *Errors in the estimation of the loudspeaker system minimum phase response are analyzed. Hilbert transforms for discrete and continuous systems are compared for minimum phase estimation. Cepstrum and system identification methods are proposed as more accurate minimum phase estimation methods in analysis of loudspeaker systems.*

INTRODUCTION

Transfer function of a continuous system can be expressed as a polynomial quotient:

$$H(s) = \frac{\sum_{i=0}^{n} b_i s^i}{\sum_{k=0}^{m} a_k s^k} = k \frac{\prod_{i=0}^{n} (s - s_{zi})}{\prod_{k=0}^{m} (s - s_{pk})} \tag{1}$$

where $s = \sigma + j\omega$ is a complex frequency, s_{zi} zeros, s_{pk} poles, n numerator polynomial order, m denominator polynomial order ($n < m$), and $k = n/m$ is a constant. Frequency response $H(j\omega)$ is given by substituting $s \rightarrow j\omega$:

$$H(j\omega) = |H(j\omega)| e^{j\varphi(\omega)} = e^{\alpha(\omega) + j\varphi(\omega)} = e^{\Theta(\omega)} \tag{2}$$

where $\Theta(\omega)$ is a logarithmic frequency response, $\alpha(\omega) = \ln|H(j\omega)|$ is a logarithmic amplitude response and $\varphi(\omega)$ is a phase response of the system. If $\ln(H(s))$ is analytic function for $\mathrm{Re}(s) > 0$, that is if there are no zeros in the right half-plane of the complex frequency plane, the system exhibits minimum phase behavior. For such system, amplitude and phase response are related by the Hilbert transform [5], and a phase of the equivalent minimum phase system $\varphi_m(\omega)$ can be computed from the logarithmic amplitude response by integral transform:

$$\varphi_m(\omega) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\alpha(\omega)}{\omega - \Omega} d\Omega = -2\alpha(\omega) \otimes \frac{1}{\omega} \tag{3}$$

Expression (3) is a convolution integral in the frequency domain. It can be calculated by using the Fourier transform technique, as follows:

$$\varphi_m(\omega) = -j\Im(\mathrm{sgn}(t)\cdot\Im^{-1}(\alpha(\omega))), \qquad \text{where} \quad \mathrm{sgn}(t) = \begin{cases} 1, t>0 \\ 0, t=0 \\ -1, t<0 \end{cases} \tag{4}$$

The design complexity of the loudspeaker system can be greatly reduced if the loudspeaker drivers behave as a minimum phase systems. For example, if loudspeaker drivers are time aligned, crossover and equalizer design can be based on the amplitude-frequency transfer function smoothing, as most of the electrical circuits are minimum phase systems. In estimating phase behavior of the loudspeaker system, after the transfer function has been measured, calculation of minimum phase according to (3) and (4) or by poles and zeros estimation [1] can be done. By comparing loudspeaker phase response with calculated minimum phase response, which corresponds to the measured amplitude response, it can be concluded in which frequency range loudspeaker driver behaves as a minimum phase system. Next we analyze which method gives the best estimation of the minimum phase response.

MINI MUM PHASE ESTI MATION USING DFT

It is common practice to estimate the minimum phase of continuous systems by calculating minimum phase from sampled impulse response $h(nT)$ ($f_s=1/T$ is a sampling frequency, n=0,l,.,N-1) ([3], [4]). By using the Discrete Fourier Transform (DFT), instead of the Fourier integral, in expression (4), minimum phase can be estimated at discrete frequencies kf_s/N, $k=0,1,2..N$. This defines the Hilbert transform for discrete systems $H(e^{jw})$, $w= 2\pi f/f_s$ [2]:

$$\varphi_m(w) = \arg\left[H(e^{jw})\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\left|H(e^{j\Omega})\right| \cot(\frac{\Omega - w}{2}) d\Omega \tag{5}$$

Using this expression for the estimation of the minimum phase response of the continuous system, gives large errors, as phase of discrete system is a periodic in frequency, with period equal to sampling frequency (see Fig. 2).

PIECEWISE HI LBERT INTEGRAL EVALUATION

Hilbert integral (3) can be more accurately calculated by the piecewise approximating logarithmic amplitude response $\alpha(\omega)$ (see Fig.1):

$$\alpha(\omega) \cong \sum_{k=0}^{n} r_k a_k(\omega), \qquad a_k = \begin{cases} 0 & \mathit{for} \quad \omega \le \omega_{k-1}, \quad a_0 = 1 \\ \dfrac{\omega - \omega_{k-1}}{(\omega_k - \omega_{k-1}} & \mathit{for} \quad \omega_{k-1} < \omega < \omega_k \\ 1 & \mathit{for} \quad \omega > \omega_k, \quad k = 1,2,..n \end{cases} \tag{6}$$

where $r_k/(\omega_k-\omega_{k-1})$ is line slope of segment $\alpha(\omega_{k-1}<\omega<\omega_k)$.

Then, minimum phase is given by the expression:

$$\varphi_m(\omega) = \sum_{k=0}^{n} r_k b_k(\omega), \qquad b_k = \frac{1/\pi}{\omega_k - \omega_{k-1}} \int_{\omega_{k-1}}^{\omega_k} \ln\left|\frac{\Omega - \omega}{\Omega - \omega}\right| d\Omega \tag{7}$$

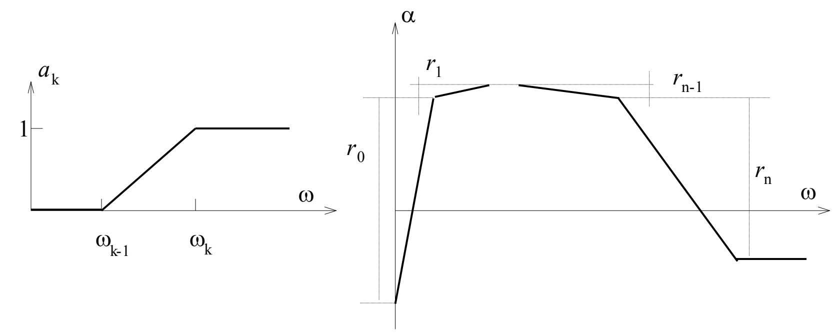

Fig. 1. Elements of logarithmic response piecewise approximation

This expression is valid only if $\alpha(\omega)$ is finite on all frequencies. Loudspeakers are "AC" coupled systems, and on both extreme frequencies loudspeaker response is zero ($\alpha \rightarrow -\infty$). Then, to get loudspeaker minimum phase response, two additional approximations must be made:

1. zero-frequency response must be held finite (this is usual in FFT analyzer)

2. frequency response above highest measured frequency have to be assumed to be constant and equal to response at this frequency, or even better approximated with a straight line of known slope.

In the Figure 2. the phase for the second order low-pass filter is compared with a minimum phase calculated by the DFT and by piecewise Hilbert integral evaluation. In both cases errors are very large, so both methods are unacceptable.

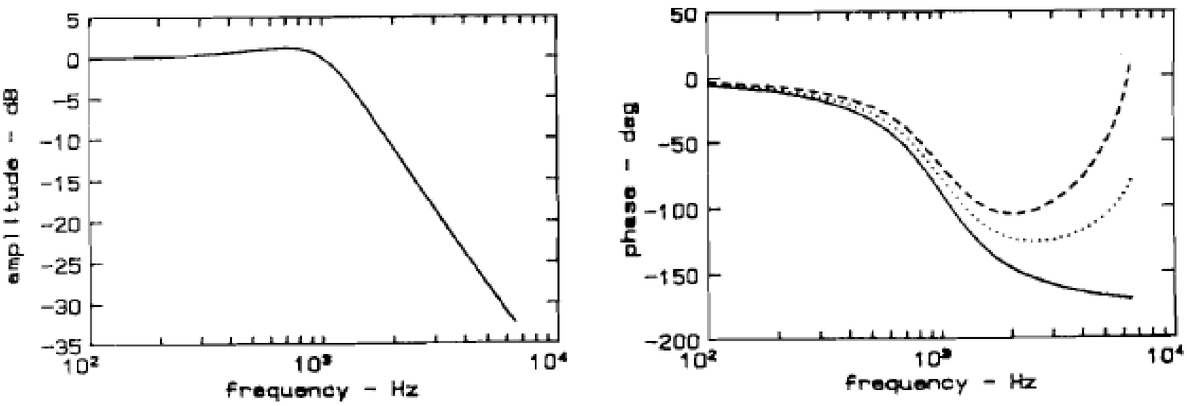

Fig. 2. a) Amplitude response of II order filter. b) minimum phase (full line). DFT-estimated (dashed) and calculated from Hilbert integral (dotted).

MINIMUM PHASE ESTIMATION AND SYSTEM IDENTIFICATION

In work [1] the method for transfer function identification is presented. System order, polynomial coefficients and poles and zeros of the transfer function are estimated from sampled impulse response by least-square error and state-space technique. This method can be applied to loudspeaker system, but some restrictions have to be encountered. Generally, loudspeaker systems are infinite order systems as delayed diffracted and reflected acoustic waves introduce delays. Delays can be approximated with cascade of finite number of all-pass transfer function $H_{di}(s)= (s-s_{di})/(s+s_{di})$, where s_{di} represents zeros in the right half plane of the complex frequency. If the estimated transfer function has M zeros with $\mathrm{Re}(s_{di})>0$ it is non-minimum phase and can be represent as product of minimum phase transfer function $H_{mp}(s)$ and an all-pass transfer function:.

$$H(s) = H_{mp}(s) \prod_{i=1}^{M} \frac{s - s_{di}}{s + s_{di}}$$

where, $H_{mp}(s)$ is equal to H(s) with all s_{di} replaced by - s_{di}.

For small system order, the transfer function estimation can be done with a high accuracy. Large delays introduce great number of all-pass sections and system order tends to be very high. Then, transfer function estimation by state-space methods [1] fails as system description matrices becomes ill conditioned. In non-anechoic measurement setup reflection from walls and loudspeaker stands contribute to these delays. Deconvolution of echoes from measured impulse response can be done by short-pass liftering complex cepstrum of impulse response [2].

Now, complete procedure for minimum phase estimation can be summarized:

a) measurement of system impulse response
b) echo deconvolution by short-pass liftering complex cepstrum of the impulse response.
c) estimation of transfer function H(s) ,
d) calculation of minimum phase according to (8).

As an example of this method, frequency response and minimum phase of unmounted dome tweeter is presented in Figure 3.

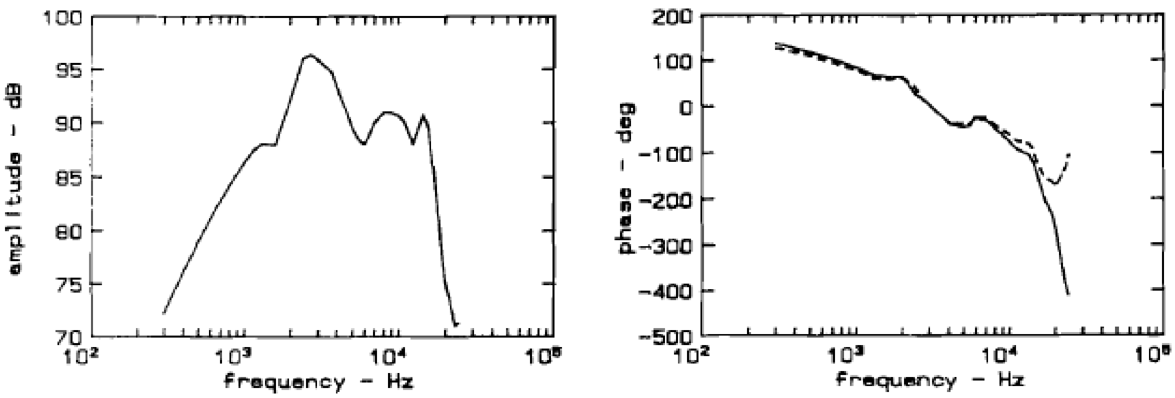

Fig. 3. a) Amplitude response. b) phase (full line) and minimum phase (dashed) of unmounted 25mm dome tweeter.

Results shown in Fig. 3 are obtained with single-channel sampled system that has AD converter with low order antialiasing filter (second order low-pass filter). Nowdays, it is common to use oversampled converters with very high order of the antialiasing filter. It additionally changes phase characteristics and makes the exact estimation of the minimum phase response impossible.

CONCLUSION

The minimum phase estimation of loudspeaker systems cannot be accurately done by the DFT-method or by numerical calculations of the Hilbert transform integral. For systems with small delays, for which the estimation of zeros and poles of the transfer function can be done, minimum phase estimation can be done more accurately.

LITERATURE

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